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THE SEQUENTIAL METHOD APPLY TO ESTIMATE THE CONVECTION HEAT TRANSFER COEFFICIENT

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ABSTRACT

This work aims to apply the zero order sequential method [1] to five models to estimate the convection heat transfer coefficient, *h*, using the transient temperature measurements. Three different regularization parameters are analyzed. The values used for regularization parameter, α, are 0.0, 1.E-3 and 1.E-7. Comparison between analytical convection heat transfer coefficient and estimated convection heat transfer coefficient are presented. Also, the analysis for analytical and experimental temperature values obtained from heat transfer coefficient estimate values is shown.

NOMENCLATURE:

A - lateral surface of body.
a - auxiliary parameter.
c_p specific heat coefficient.
$h(t)$ - time dependent heat transfer coefficient.
h^* - dimensionless heat transfer coefficient.
h_0 - initial value of heat transfer coefficient.
h_∞ - steady state heat transfer coefficient.
m - auxiliary parameter.
r - futures times number.
$\Re^1$ - one dimensional space.
S - functional number.
t - time.
t_0 - initial value time.
T_0 - initial value temperature.
T_∞ - steady state temperature.
$T(t)$ - time dependent temperature.
T^* - dimensionless temperature.
T_0^* - initial dimensionless temperature.
U - random set values.
Y - experimental values temperature.
$\forall$ - volume.
α - regularization parameter.
β_i - identified coefficient.
η - constant parameter in case 1.
σ - standard deviation.
ρ - specific mass.
θ - difference between T_0 and T_∞.

INTRODUCTION

In many heat transfer problems in engineering is difficult to establish the heat transfer coefficients, for example, in the conduction diffusivity coefficient, convection the heat transfer coefficient and radiation extinction coefficient. Furthermore, if the problems are governed by elliptic partial differential equations, the difficult to establish them should be very large [2].

Colaço and Orlande [3] showed an application for the conjugate gradient method to estimate (convection heat transfer coefficient), *h,* in $\Re^1$ space. Scott and Beck [4] applied the sequential regularization solutions in inverse heat conduction problem. In this work the zero order sequential method approach is used to estimate the convection heat transfer coefficient, *h*.

Five different cases for a body of known dimensions that is submitted to the temporal variation of the temperature, in $\Re^1$ space are analyzed.

The first test case for *h(t)* are considered like a constant value for heat transfer coefficient, $h(t) = h_0$. In the second test case, the variation is given by an exponential function [5]. In the third case h(t) has a variation with quadratic polynomial function, with coefficients β_i proposed by Beck et al. [6]. For the fourth case, $h(t)$ has a dependency of temperature profile and the last case has a combined function compose by exponential and a power [6].

A comparative analysis between the analytical and estimate of heat transfer coefficient and temperature for the diverse cases is shown, function of the regularization parameter, α.

ANALITIC SOLUTION

The analytic solution for the problem given by Newton's cooler law, being a first-order differential equation [7]:

$$\rho\, c_p \forall \frac{dT}{dt} = h(t) A (T_\infty - T(t)) \quad (1)$$

With initial condition: $T(t_0) = T_0$

That is, to found the time dependent temperature, $T(t)$, it is necessary before know the time dependent heat transfer coefficient $h(t)$.

ZERO ORDER SEQUENTIAL METHOD

This method minimizes the differences between estimated and real temperatures adding a regularization term [1]. Beck developed zero order sequential method for flux problems in inverse heat conduction, but in this work it is utilized the regularization parameter, α applied to estimate heat transfer coefficient, h_{M+i-1} .

In this particular application is assumed the quadratic sum for h coefficients, given by:

$$S = \sum_{i=1}^{r} (Y_{M+i-1} - T_{M+i-1})^2 + \alpha \sum_{i=1}^{r} h_{M+i-1}^2 \quad (2)$$

The identical approach applied in conduction problem, has presented by Woodbury and Ke [8], when the function $h(t)$ was set constant and the value by $h(t_M) = h_M$, for the time t_M, is determined for t_{M+1}. At each time step, r futures times of data are used to regularize the problem. For evaluate the results by h_{M+i-1} are calculated the values by temperature T_{M+i-1} across the relation:

$$T_{M+i-1} = T_{M-1} \exp\left(-m h_M \left(t_{M+i-1} - t_{M-1}\right)\right) \quad (3)$$

The value of h_M is temporarily held constant over the r time steps.

Differentiating the equation (2) with respect to h_M and setting this derivate equal to zero, the result is:

$$\alpha h_M - \sum_{i=1}^{r} (Y_{M+i-1} - T_{M+i-1}) Z_{M+i-1} = 0 \quad (4)$$

Differentiating the temperature with respect to h_M and using the interactive algorithm proposed by Beck et al. [1]:

$$h_M^{\nu} = \frac{\sum_{i=1}^{r} \left(Y_{M+i-1} - T_{M+i-1}^{\nu-1} + h_M^{\nu-1} Z_{M+i-1}^{\nu-1}\right) Z_{M+i-1}^{\nu-1}}{\alpha + \sum_{i=1}^{r} \left(Z_{M+i-1}^{\nu-1}\right)^2} \quad (5)$$

The superscript ν indicates the interaction number and Z is the sensitivity coefficient.

METHODOLOGY

The procedure consists in:

Step 1: The *h(t)* profile, for different cases are generated.

Step 2: Using the analytical profile by *h(t),* test case, to find $T(t)$, (analytical temperature) solving equation (1).

Step 3: Calculated $T_{M+i-1}; Z_{M+i-1}; h_{M+i-1}$ and S to five different models used in the algorithm of heat transfer estimation. The temperatures are compared between analytical and experimental heat transfer coefficients.

TEST CASES

Case 1

In this in case, the convection heat transfer coefficient are considered constant, assuming $h(t) = 10\ \text{W/m}^2\text{K}$, a constant function. A parameter η_1 is defined as:

$$-\eta_1 = h_0 A / \rho\, c_p \forall \quad (6)$$

The solution of equation (1) is:

$$T(t) = T_\infty + (T_0 - T_\infty).\exp(-\eta_1 t) \quad (7)$$

Using dimensionless variable and changing parameters in equation (1):

$$T^* = \frac{T_\infty - T}{T_\infty - T_0}; \quad h^* = \frac{h(t)}{h_\infty}; \quad a = \frac{h_0}{h_\infty}; \quad m = \frac{A.h_\infty}{\rho.cp.\forall} \quad (7)$$

$$\frac{dT^*}{dt} + mh(t)T^* = 0 \quad (8)$$

The solution is:

$$T^* = T_0^*.\exp\left(-m.h^*.t\right) \quad (9)$$

Figure (1) shows the comparative analysis between analytical and experimental temperature values obtained from Eq.(3) and Eq.(5).

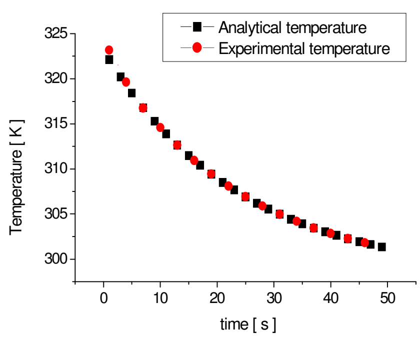

Figure 1 – Temperature profile, Case1.

In Fig. (1), it should be seen that the differences are very small among analytical and experimental temperature values. Experimental temperature values are obtained Eq.(3) by an interactive algorithm, where the previous interaction (ν) is necessary to solve the next interaction (ν+1).

Figure (2), presents the analysis between analytical heat transfer coefficient and estimate values for all regularization parameters. The differences between h estimate values are very small approximated in third decimal order as Fig. (2).

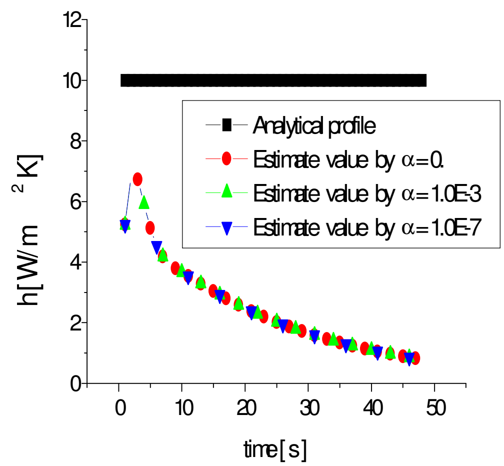

Figure 2 – Estimation to heat transfer coefficient.

An analysis by the functional S, Eq.(2), Tab.(1) showed that the best regularization parameter is α= 0. For α= 1E-7 the difference is a fifth decimal order and for α = 1E-3 the S values are lass then α= 0 and α= 1E-7 as showed in Fig. (3)

Table 1- S values by tree regularization parameters in Case1.

ALFA	S
0.0	**4,9817769157**
1.0E-3	**5,3169177400**
1.0E-7	**4,9817945646**

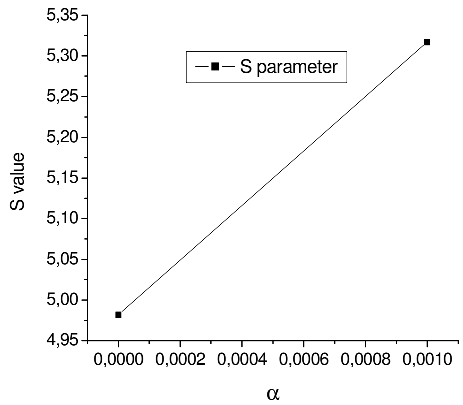

Figure 3– Functional analysis for α values.

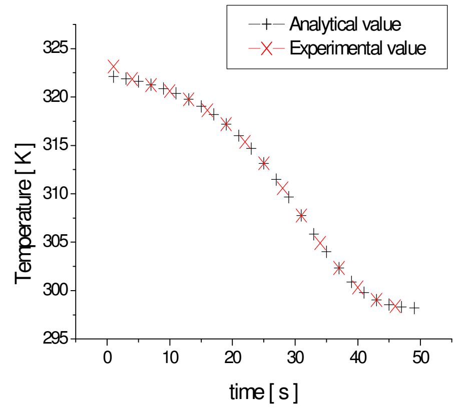

Figure 4 – Analytical and experimental temperature values.

Case 02

In this case the heat transfer coefficient, $h(t)$, is written as an exponential profile, as in the work by Cardoso, et al. [5], given by:

$$h(t) = h_\infty + (h_0 - h_\infty).\exp(-\beta_1 t) \tag{10}$$

Solve the Eq.(1) with *h(t)* profile it is found *T(t)* profile given by:

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left[h_\infty \frac{A}{\rho c_p \forall} t + \left(\left(\frac{-A}{\rho c_p \forall b}\right)\exp(-bt)\right)\right] \tag{11}$$

The dimensionless parameters are defined as in case one:

$$T^* = C.\exp\left|-m.t + \frac{m.(a-1)}{b}.\exp\left|-\beta_1.t\right|\right| \tag{12}$$

With :

$$C = T_0.\exp\left|mt_0 - \frac{m.(a-1)}{b}.\exp\left|-\beta_1.t_0\right|\right|$$

The absolute differences among temperature values are third decimal order as showed in Fig.(4).

Figure 5 shows the analysis between *h(t)* values and estimate heat transfer coefficient values for the regularization parameters.

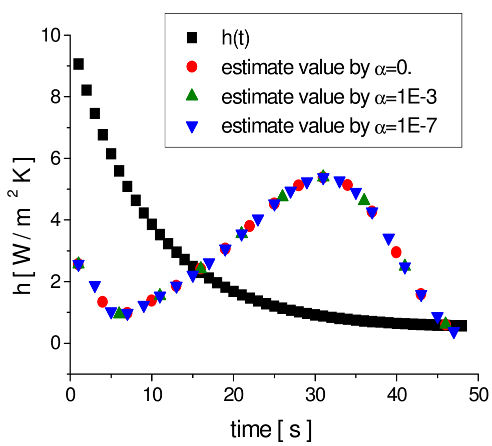

Figure 5 – h estimate values.

Tab.(2) present the values for functional S in Case 2 for regularization parameters, α.

Table 2 – S values by regularization parameter values in case2.

ALFA	S
0.0	*6,453576*
1.0 E –3	*4,868277*
1.0 E -7	*4,868126*

Figure (6) present the values described in Tab.(2) for regularization parameter α applied in case 2.

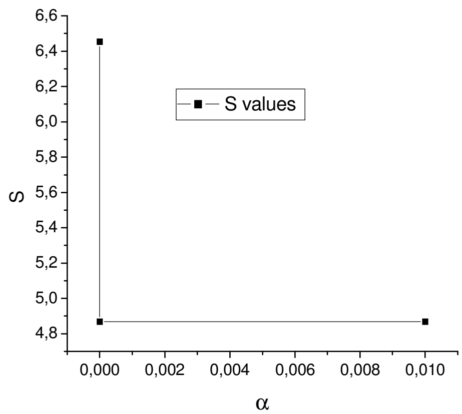

Figure 6 – S values for α in case2.

Case 03

In this case the heat transfer coefficient, $h(t)$, is written as a quadratic function [6], given by:

$$h(t) = \frac{A}{\rho\ cp\ \forall}\left[\beta_1 + \beta_2\, t + \beta_3\, t^2\right] \qquad (13)$$

The analytical temperature profile is given by:

(14)

Utilizing dimensionless changes:

$$T^* = T_0^*\, .EXP\left[\frac{m}{h_\infty}(\beta_1\, t + \frac{\beta_2}{2} t^2 + \frac{\beta_3}{3} t^3\right] \qquad (15)$$

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left[-(\beta_1 t + \frac{\beta_2}{2} t^2 + \frac{\beta_3}{3} t^3)\right]$$

Figure (7) shows the comparison between analytical and experimental values by temperature from the algorithm the estimation for heat transfer coefficient in case 3.

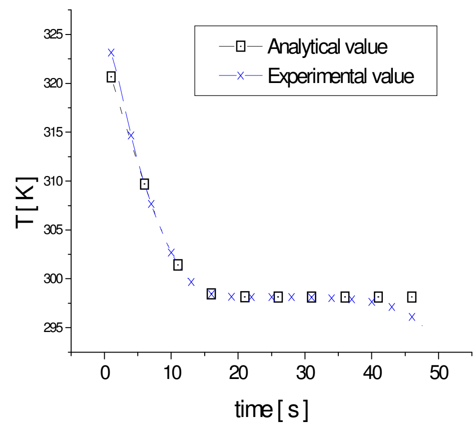

Figure 7– Comparison between temperature values.

Figure (8) presented the comparison between the analytical profile by *h(t)* and the estimate values for the heat transfer coefficient . In this case the physical model not is respected because assuming the quadratic function for *h(t)* the body present a positive variation for h, or either, heating. The estimate values presents in Fig. (8) showed the behavior according to physical model to the body cooling.

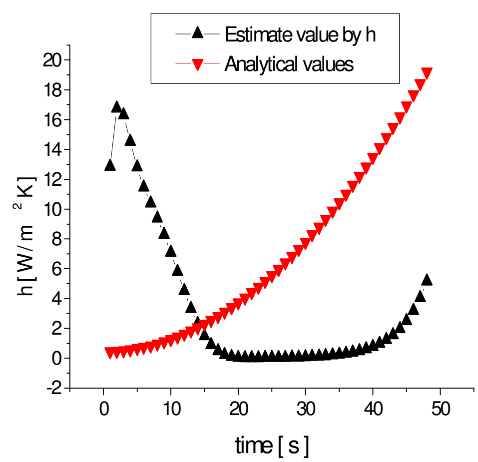

Figure 8 – Comparison among estimate and analytical suppose values by h.

In This case the best regularization parameter is α = 1E-3 because with the errors in analytical model, the zero order method applied the minimize in this errors as showed in Fig. (9) and in table (3).

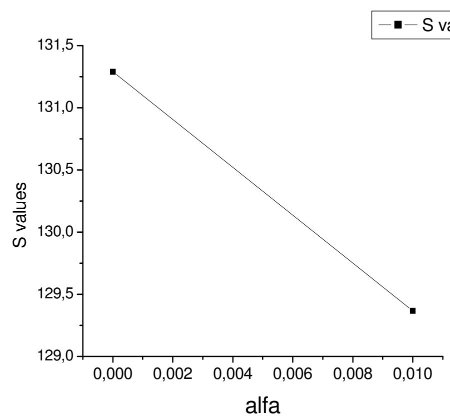

Figure 9 – S values in case3

Table 3- S values for Case3 to α values.

ALFA	S
0.0	131,2900737
1.0 E -3	129,3676299
1.0 E -7	131,2898973

Case 04

In this case $h(t)$ is given by [6]:

$$h(t)=\frac{\rho\, c_p\, \forall}{A}\left[\beta_1+\beta_2\left(T(t)-T_\infty\right)\right] \tag{15}$$

and the solution of the equation (1) is:

$$T(t)=T_\infty+(T_0-T_\infty)$$
$$\left\{\exp(-\beta_1 t)\,\left[1+\frac{\beta_2}{\beta_1}(T_0-T_\infty)\,\left(1-\exp(-\beta_1 t)\right)\right]\right\}^{-1} \tag{16}$$

Using of dimensionless variables:

$$T^*=T_0^*\,EXP\left[m\,t\left(\frac{\beta_1+\beta_2\left(T(t)-T_\infty\right)}{h_\infty}\right)\right] \tag{17}$$

In this case the same tendency is showed for previous cases to analytical and experimental temperature values as can see in Fig.(10).

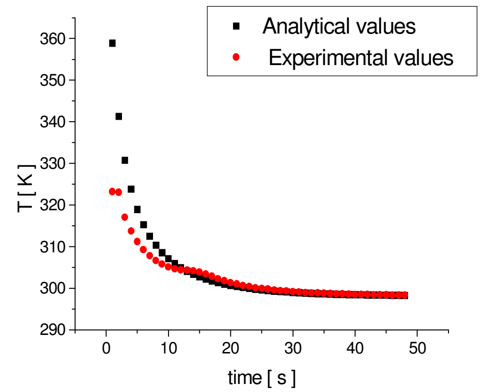

Figure 10 - Comparison among analytical and experimental values for temperature.

Then the analysis by heat transfer coefficient the Fig.(11) presented a comparison among analytical and estimate values for heat transfer coefficient.

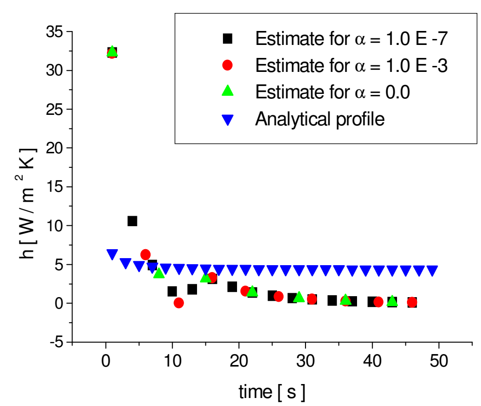

Figure 11 – h estimate values for α values.

In This case the best regularization parameter is α = 1E-3 because the difference between in analytical model and experimental values, the zero order method are important to minimize this errors. Fig.(12) and in Tab. (4).

Table (4) – S values for α.

ALFA	S
0.0	*4964,078*
1.0 E –3	*4903,025*
1.0 E -7	*4964,072*

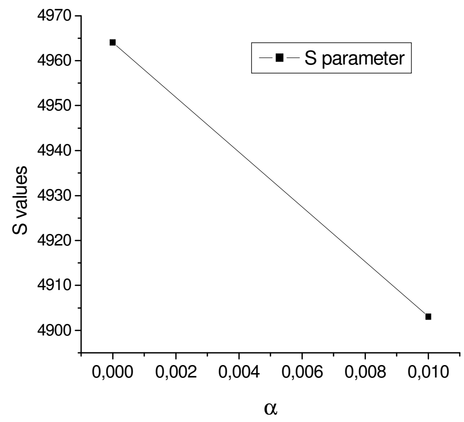

Figure 12 – S values in case four.

Case 05

In this case $h(t)$ is written as a composition of two function: exponential profile and power.

$$\theta = T_0 - T_\infty$$

$$h(t) = T_\infty + \theta . \left\{ \exp(-\beta_1 t) \left[1 + \frac{\beta_2}{\beta_1} \theta \left(1 - \exp(-\beta_1 t) \right) \right]^{-1} \right\} \qquad (18)$$

equation (1) has the following solution for this case:

$$\theta = T_0 - T_\infty$$

$$T(t) = T_\infty + \theta . \exp\left\{ \frac{A}{\rho\, cp \forall} \theta \left\{ \exp(-\beta_1 t - 1) \left(\frac{-1}{\beta_1} - \frac{\beta_2}{\beta_1^2} \theta \right) \right\} \right\} \qquad (19)$$

Using dimensionless variables:

$$\theta = T_0 - T_\infty$$

$$T^* = T_0^* \exp\left\{ -m \left[\left(\frac{T_\infty}{h_\infty} t \right) + \frac{\theta}{h_\infty} \left\{ \exp(-\beta_1 t - 1) \left(\frac{-1}{\beta_1} - \frac{\beta_2}{\beta_1} \theta \right) \right\} \right] \right\} \qquad (20)$$

In Fig. (13) shows the small difference among analytical temperature profile and experimental temperature values obtained from estimate heat transfer coefficient values in Eq. (3) by an interactive algorithm.

Figure (14) presents a comparison among the values for heat transfer coefficient estimate and analytical applied in case five.

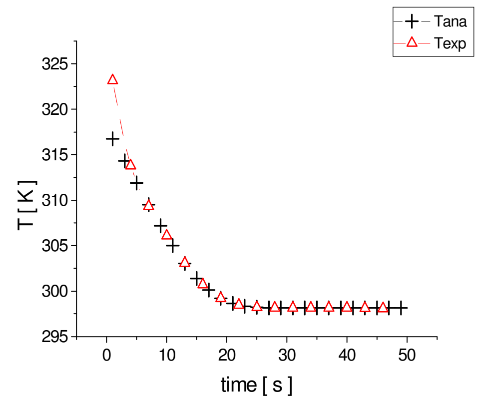

Figure 13 – Comparison among analytical and experimental temperature values.

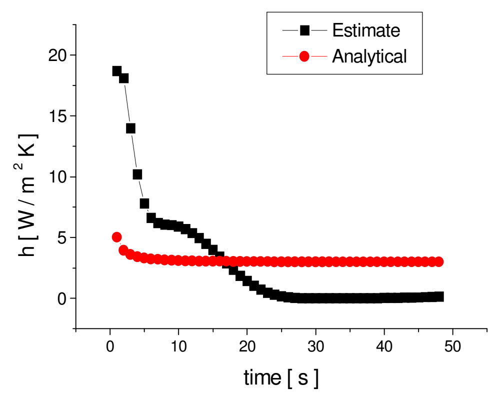

Figure 14 - Comparison among analytical and experimental heat transfer coefficient values.

The best value by regularization parameter is presented in Tab.(5) to Case 5.

Table 5 – S values for α in case five.

ALFA	S
0.0	*88,856359*
1.0 E –3	*92,374490*
1.0 E -7	*88,856359*

The values by functional S are presents in Fig. (15) where to α =0.0 and α = 1.0 E –7 the functional S has a equal value.

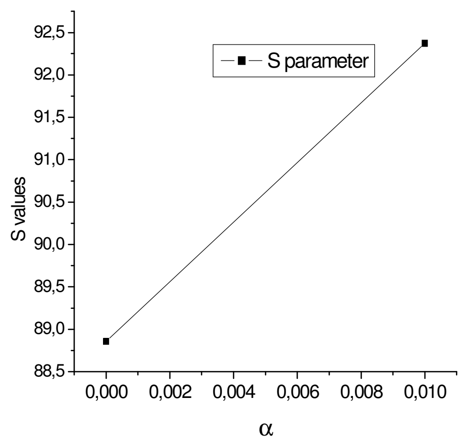

Figure 15 – S values by α in Case5.

CONCLUSION

Five cases to estimate the convection heat transfer coefficient was analyzed for cooling billet model, using the zero order regularization sequential method.

Case 1 was assumed that heat transfer coefficient is constant in practical problem the heat transfer coefficient has a behavior as Fig. (2). In Cases 1 and 2 the best value obtained for regularization parameter, α is zero. For all cases analyzed the values for analytical and experimental temperatures the difference is very small as in Figs. (1);(4); (7) and (13).

Case 3 although the temperature shows similar to analytical temperature, the identified heat transfer coefficient has a different profile. In this case the zero order method applied execute the correction by h estimate values with robustness form.

Cases 4 and 5 satisfactory because the differences among analytical and experimental values by temperature are very small as shown in Fig.(13). Then the heat transfer analysis as shown in Fig.(14) the same tendency among the values.

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